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GROUP-THEORETICAL ANALYSIS OF THE CLUSTERED LAUNCH VEHICLE DYNAMICS

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Abstract

In this paper we considered representation-theory-based eigenfunction classification of clustered launch vehicles vibration problems. Classification of vibrations modes was obtained by using projection operators, related with corresponding subspaces of irreducible representations of considered mechanical system symmetry group. For multiple frequencies we proposed the approach which allows to reduce corresponding vibrations modes to launch vehicle stabilization planes. In addition, for the launch vehicle with four boosters, the projections onto irreducible representations subspaces of right-hand side of the motion equations were found

Keywords

Launch vehicle, beam system, symmetry group, vibration frequency, irreducible representation, orthoprojector

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Introduction. In the analysis of launch vehicle dynamics, the method when the solution of corresponding evolutionary problem is decomposed by natural vibration modes is widely used [1]. In the case of the tandem launch vehicle, vibration modes can be classified into longitudinal, bending and torsional. As a result, the analysis of launch vehicle motion can be performed separately according to guidance system channels, by the assumption that different abovementioned types of vibration do not affect each other. The longitudinal vibrations of a clustered launch vehicle were investigated in [2–4].

In case of a clustered launch vehicle, the utilization of natural vibration modes in dynamic processes analysis becomes much more complicated, due to launch vehicle undergoes joint longitudinal-bending-torsional vibrations, and thereby, the assignment of any given vibration mode to the considered control loop is not univocal. In addition, the eigenvalues spectrum of the corresponding spectral problem (vibration frequencies) is much denser than for a tandem launch vehicle (several dozens of frequencies less than 10 Hz for a heavy launch vehicle), therefore the use of the entire set of vibration modes greatly complicates the problem.

Works [5–8] give a classification of vibrations modes of a clustered launch vehicle, based on possible displacements of the core stage. In this paper, using a clustered launch vehicle with four boosters as an example, we propose a metedology of vibration modes classification for clustered launch vehicles with an arbitrary number of boosters based on the symmetry of the mechanical system being investigated. In our approach we use the representation theory of finite groups of symmetry transformations. A similar approach was previously used in studies of other mechanical systems having symmetry [9–14].

Problem statement. The beam system is used as the analytical model. The elastic elements will simulate spider beams which hold boosters. Transverse and axisymmetric fluid sloshing in tanks, as well as vibrations of structural elements (propulsion system, pipelines, etc.), if necessary, can be taken into account with the help of an oscillator analogy. Since the movements of oscillators are unambiguously expressed through the beams displacements, and if their inclusion into mechanical system does not break the symmetry, it follows that symmetry classification of vibration modes will not change as well.

The Fig. 1 shows the beam model of a clustered launch vehicle. This mechanical system consists of a central beam and identical side beams connected to the central one by linear-elastic connectors. The central beam is located along the X_0 axis of the global coordinate system $X_0Y_0Z_0$, and the end of the beam corresponding to the lower part of the rocket coincides with its origin. The side beams are located in parallel to the central one at an equal distance from it, through angular intervals equal to $2\pi / N$ (N is the number of side beams). The side beams correspond to the indices j, and the beam with index 1 placed in the X_0Y_0 plane, the indexing of the beams is increased counterclockwise around the axis X_0. The lower ends of the side beams are located in one plane parallel to Y_0Z_0. Further, we assume that this plane coincides with Y_0Z_0.

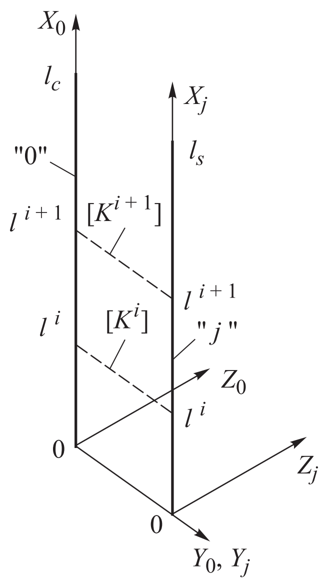

Fig. 1. The beam model of a clustered launch vehicle

A local coordinate system $X_jY_jZ_j$ corresponds to each side beam. A coordinate origin coincides with the lower end of the side beam, the Y_j axis is directed at an angle of $2\pi\,(j - 1) / N$ to the Y_0 axis, the X_j axis is parallel to the X_0 axis, and the Z_j axis complements the coordinate system to the right-handed one.

The displacements of the sections of beams along the X_j, Y_j and Z_j axes are denoted as $u_j\,(x, t)$, $v_j\,(x, t)$ and $w_j\,(x, t)$, respectively, where $j = 0, \ldots, N$. Since the origins of the local coordinate systems lie in the Y_cZ_c plane, it is possible not to distinguish between the coordinate systems of the beams in the spatial argument of the displacement functions, which can be denoted as x. The angle of rotation of the sections around the X_j axes is denoted as $\psi_j(x, t)$, $j = 0, ..., N$.

It is considered that vibrations of one type do not affect the other and are described by separate differential equations. The connection of the longitudinal, bending and torsional vibrations of the beams will be considered through the boundary and blending conditions. We will not take into account the inertia of the section during rotation and the Timoshenko shear coefficient for bending vibrarions (we will consider the Euler — Bernoulli beam).

Finite symmetry groups of the spectral problem generated by the clustered launch vehicle vibrations. The design features of the clustered rocket in most cases lead to the fact that the corresponding mechanical model turns out to be symmetrical with respect to turns at angles multiple of the angular distance between adjacent side blocks and reflections relative to the planes passing through the axis of the core stage and the axis of the boosters as well as axis dividing the resulting dihedral angles in half. Such launch vehicles include various modifications of the Angara, Soyuz, Delta-IV Heavy, Falcon Heavy, Ariane-5,6, CZ-3B, 3C, 5, GSLV, etc. The exception is the Energia launch vehicle and some modifications of the Ariane 4, PSLV and similar rockets that have a smaller set of symmetries due to differences in the boosters and their location.

According to Shenflis, such groups of spatial symmetry are denoted as C_{nv}. The symmetry transformations that form them allow rotations of C_n by angles that are multiples of $2\pi / n$, and reflections with respect to n planes of symmetry located across π / n angular intervals. Further, the transformations of these groups will be described in the global coordinate system $X_0Y_0Z_0$. Rotations are understood as active rotations, i.e., the object rotates itself, but the coordinate system remains fixed.

Launch vehicle with four boosters. Let us consider launch vehicle with four booters. A corresponding symmetry group is C_{4v}. This group includes eight elements: E, C_2, C_4, C_4^{-1}, σ_Y, σ_Z, σ_1, σ_2. Here, σ_Y is the reflection relative to the plane X_0Y_0, σ_Z is the reflection relative to the plane X_0Z_0, σ_1 and σ_2 are reflections relative to the planes rotated $\pi / 4$ from σ_Y and σ_Z. The C_{4v} group has five irreducible representations (see table below) [15].

The table contains the following notation: σ_v denotes σ_Y and σ_Z while σ_d denotes σ_1 and σ_2. The displacement vector in this case contains 20 components and can be written as follows:

$$U = \{u_0, u_1, u_2, u_3, u_4, v_0, v_1, v_2, v_3, v_4, w_0, w_1, w_2, w_3, w_4, \psi_0, \psi_1, \psi_2, \psi_3, \psi_4\}^t.$$

In order to find orthoprojectors onto one-dimensional representations, we use the following equation:

$$P^{(R_\varphi)} = \frac{1}{G(C_{4n})} \sum_{C_{4n}} \chi^{(R_\varphi)}(g) T(g),$$

and onto lines of 2D representation E

$$P^{(E^i)} = \frac{2}{G(C_{4n})} \sum_{C_{4n}} T_{ii}^{(E)}(g) T(g), \quad i = 1, 2.$$

Characters of group irreducible representations C_{4v}

Irreducible representations	Group elements C_{4v}				
	E	C_2	$C_4(2)$	$\sigma_v(2)$	$\sigma_d(2)$
A_1	1	1	1	1	1
A_2	1	1	1	–1	–1
B_1	1	1	–1	1	–1
B_2	1	1	–1	–1	1
E	2	–2	0	0	0

Here $G(C_{4v})$ is the C_{4v} group order; $\chi^{(R_\varphi)}(g)$ are one-dimensional irreducible representations characters; $T(g)$ are C_{4v} group representarion operators; $T_{ij}^{(E)}$ are diagonal matrix elements of an irreducible representation E.

The expressions for orthoprojectors on irreducible representations of a C_{4v} group are written as follows:

on one-dimensional irreducible representations

$$P^{(A_1)} = \frac{1}{8}\Big(1 \cdot T(E) + 1 \cdot T(C_2) + 1 \cdot T(C_4) + 1 \cdot T(C_4^{-1}) + 1 \cdot T(\sigma_Y) + \\ + 1 \cdot T(\sigma_Z) + 1 \cdot T(\sigma_1) + 1 \cdot T(\sigma_2)\Big);$$

$$P^{(A_1)} = \frac{1}{8}\Big(1 \cdot T(E) + 1 \cdot T(C_2) + 1 \cdot T(C_4) + 1 \cdot T(C_4^{-1}) - 1 \cdot T(\sigma_Y) - \\ - 1 \cdot T(\sigma_Z) - 1 \cdot T(\sigma_1) - 1 \cdot T(\sigma_2)\Big);$$

$$P^{(A_2)} = \frac{1}{8}\Big(1 \cdot T(E) + 1 \cdot T(C_2) + 1 \cdot T(C_4) + 1 \cdot T(C_4^{-1}) - 1 \cdot T(\sigma_Y) - \\ - 1 \cdot T(\sigma_Z) - 1 \cdot T(\sigma_1) - 1 \cdot T(\sigma_2)\Big);$$

$$P^{(B_1)} = \frac{1}{8}\Big(1\cdot T(E) + 1\cdot T(C_2) - 1\cdot T(C_4) - 1\cdot T(C_4^{-1}) + 1\cdot T(\sigma_Y) + \\ +1\cdot T(\sigma_Z) - 1\cdot T(\sigma_1) - 1\cdot T(\sigma_2)\Big);$$

$$P^{(B_2)} = \frac{1}{8}\Big(1\cdot T(E) + 1\cdot T(C_2) - 1\cdot T(C_4) - 1\cdot T(C_4^{-1}) - 1\cdot T(\sigma_Y) - \\ -1\cdot T(\sigma_Z) + 1\cdot T(\sigma_1) + 1\cdot T(\sigma_2)\Big);$$

on the 1st and 2nd lines of an irreducible representation E

$$P^{(E^1)} = \frac{2}{8}\Big(1\cdot T(E) - 1\cdot T(C_2) + 0\cdot T(C_4) + 0\cdot T(C_4^{-1}) + 1\cdot T(\sigma_Y) - \\ -1\cdot T(\sigma_Z) + 0\cdot T(\sigma_1) + 0\cdot T(\sigma_2)\Big);$$

$$P^{(E^2)} = \frac{2}{8}\Big(1\cdot T(E) - 1\cdot T(C_2) + 0\cdot T(C_4) + 0\cdot T(C_4^{-1}) - 1\cdot T(\sigma_Y) + \\ +1\cdot T(\sigma_Z) + 0\cdot T(\sigma_1) + 0\cdot T(\sigma_2)\Big).$$

Solving the equations $P^{(R_\varphi)}U = U$ for each irreducible representation (lines of irreducible representations), we can write the general form of the basis vectors of irreducible representations of the C_{4v} group:

$$\begin{aligned}
U^{(A_1)} &= \{u_0, u_s, u_s, u_s, u_s, 0, v_s, v_s, v_s, v_s, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}^t;\\
U^{(A_2)} &= \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, w_s, w_s, w_s, w_s, \psi_0, \psi_s, \psi_s, \psi_s, \psi_s\}^t;\\
U^{(B_1)} &= \{0, u_s, -u_s, u_s, -u_s, 0, v_s, -v_s, v_s, -v_s, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0\}^t;\\
U^{(B_2)} &= \{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, w_s, -w_s, w_s, -w_s, 0, \psi_s, -\psi_s, \psi_s, -\psi_s\}^t;\\
U^{(E^1)} &= \{0, u_s, 0, -u_s, 0, v_0, v_s, 0, -v_s, 0, 0, 0, -w_s, 0, w_s, 0, 0, -\psi_s, 0, \psi_s\}^t;\\
U^{(E^2)} &= \{0, 0, u_s, 0, -u_s, 0, 0, v_s, 0, -v_s, w_0, w_s, 0, -w_s, 0, 0, \psi_s, 0, -\psi_s, 0\}^t.
\end{aligned} \qquad (1)$$

From expressions (1) it follows that the obtained vectors have a much smaller dimension than the dimension of the original space (the dimension was decreased from 4 to 10 times). The Fig. 2 schematically shows the modes corresponding to the vectors (1).

The vibration modes corresponding to the vectors $U^{(A_1)}$ are longitudinal vibrations of the launch vehicle, corresponding to the vectors $U^{(A_2)}$ are torsional vibrations. In the case of vectors $U^{(B_1)}$ and $U^{(B_2)}$, the perturbation acting from the side blocks on the central one is compensated, as a result we obtain that

$$u_0 = v_0 = w_0 = \psi_0 \equiv 0.$$

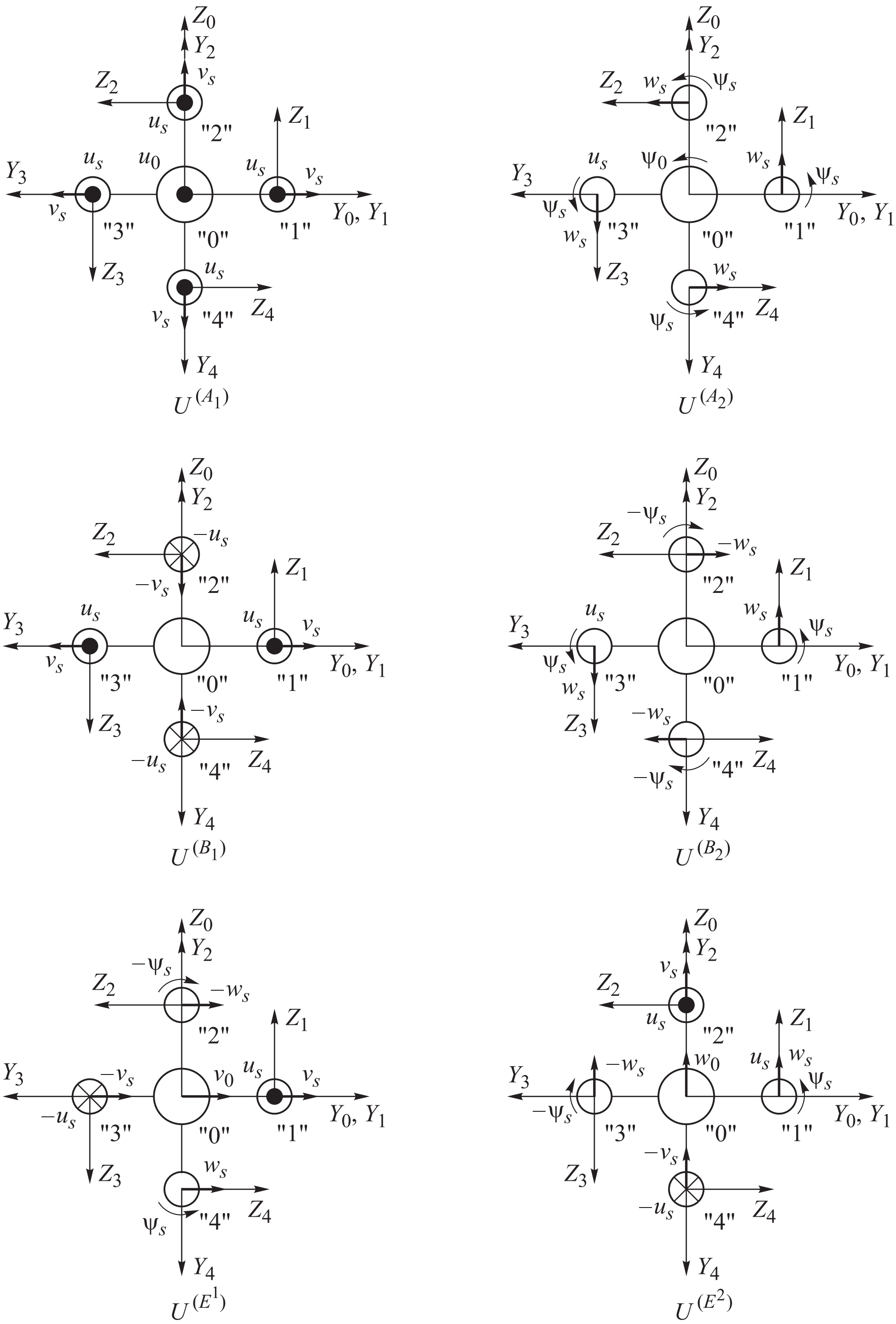

Fig. 2. Schematic representation of vibrations modes corresponding to various irreducible C_{4v} group representations

The vectors $U^{(B_1)}$ correspond to the longitudinal-bending vibrations of the side beams (the so-called barrel-shaped vibrations), the vectors $U^{(B_2)}$ are the bending-torsional vibrations of the side beams. Twice multiple frequency corresponds to the two-dimensional irreducible representation *E*. These vibrations correspond to the bending vibrations of the launch vehicle in two perpendicular planes X_0Y_0 и X_0Z_0.

Vibrations of the launch vehicle in two perpendicular planes X_0Y_0 и X_0Z_0.

For the components of displacement vectors belonging to the irreducible representation of *E*, we can write additional relations based on its matrix elements:

$$\begin{aligned} v_0^{(E^1)} &= w_0^{(E^2)}; \\ u_s^{(E^1)} &= u_s^{(E^2)}; \\ v_s^{(E^1)} &= v_s^{(E^2)}; \\ w_s^{(E^1)} &= w_s^{(E^2)}; \\ \psi_s^{(E^1)} &= \psi_s^{(E^2)}. \end{aligned} \qquad (2)$$

The launch vehicle configuration with four boosters, considered in this section, corresponds to the “+” configuration. In practice, the “×” configuration takes place as well, and can be obtained by rotating the global coordinate system $X_0Y_0Z_0$ by an angle of $\pi / 4$. For this configuration, irreducible representations basis vectors can be obtained using this technique after constructiong the corresponding representation operators.

Reduction of vibration modes corresponding to multiple frequencies to stabilization planes. Difficulties of an interpretation and further utilization of vibration modes corresponding to multiple frequencies arise if natural vibrations problem of symmetrical mechanical systems was treated by numerical methods. In the case of the considered in this paper clustered launch vehicle with four boosters, multiple frequencies correspond to modes which form basis of the irreducible representation *E*. These frequencies have a multiplicity equal to two and their eigensubspace spanned on two basic vectors. From Fig. 2, *d* and *e* it follows that the vibration modes belonging to the lines of the irreducible representation *E* are vibrations, which can be characterized as bending vibrations of the vehicle in the planes X_0Y_0 and X_0Z_0 which, in turn, coincide with the rocket stabilization planes.

At the same time, in numerical solution, for example, using the iterative Lanczos method, if we do not use additional boundary conditions (assumptions), the result of calculating these vibration modes will not be univocal, but will be determined by the first approximation specified in the corresponding iterative process. As a result, the obtained modes may not correspond to the lines of the irreducible representation of E and, therefore, to the rocket stabilization planes. From the spectral theory [16], this is explained by the fact that the linear combination of the basis vectors of the eigensubspace is also a solution of the spectral problem, and belongs to this eigensubspace. Obviously, this should be a linear transformation preserving the scalar product, i.e., orthogonal.

Let us denote the vectors obtained by the numerical calculation as $U^{(1)}$ and $U^{(2)}$, and express them through the vectors belonging to the lines of the irreducible representation E:

$$\begin{Bmatrix} U^{(1)} \\ U^{(2)} \end{Bmatrix} = \begin{pmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix} \begin{Bmatrix} U^{(E^1)} \\ U^{(E^2)} \end{Bmatrix}. \tag{3}$$

To construct the eigenvectors corresponding to the stabilization planes and the lines of the irreducible representation E, it suffices to have one vector obtained by numerical calculation. Consider a vector

$$U^{(1)} = \alpha_{11} U^{E^1} + \alpha_{12} U^{(E^2)}.$$

Alternately acting on the vector $U^{(1)}$ with orthoprojectors $P^{(E^1)}$ and $P^{(E^2)}$, we can obtain:

$$P^{(E^1)} U^{(1)} = \alpha_{11} U^{(E^1)};$$

$$P^{(E^2)} U^{(1)} = \alpha_{12} U^{(E^2)},$$

finally

$$U^{(E^1)} = \frac{1}{\alpha_{11}} \left(P^{(E^1)} U^{(1)} \right);$$

$$U^{(E^2)} = \frac{1}{\alpha_{12}} \left(P^{(E^2)} U^{(1)} \right).$$

Coefficients α_{11} amd α_{12} can be determined from the equation system

$$\begin{cases} \alpha_{11}^2 + \alpha_{12}^2 = 1; \\ \dfrac{\alpha_{11}}{\alpha_{12}} = \dfrac{v_0^{(1)}}{w_0^{(1)}}, \end{cases} \tag{4}$$

where $v_0^{(1)}$ and $w_0^{(1)}$ are corresponding components of the vector $U^{(1)}$.

Decomposition of external perturbation vector into irreducible representations of the symmetry group for clustered launch vehicle. Let longitudinal instability (for example thrust pulsation) occurs in the booster with index $j = 1$. The vector $F(t)$ in this case will contains only one component:

$$\begin{aligned} &F(t) = \left\{ F_u(t),\ F_{vw}(t),\ F_{\psi}(t) \right\}^t; \\ &F_u(t) = \{0, f(t), 0, 0, 0\}^t; \quad F_{vw}(t) = \{0\}; \quad F_{\psi}(t) = \{0\}. \end{aligned} \tag{5}$$

To decompose the vector $F(t)$ into irreducible representations of the group C_{4v}, we use expressions for orthoprojectors $P^{(R_{\varphi})}$. Further we will consider the block $F_u(t)$, since only this block will contain nonzero components. Denoting $f(t)$ as f, as a result of projection we get:

$$\begin{gathered} F_u^{(A_1)}(t) = P^{(A_1)} F_u(t) = \left\{0,\ f/4,\ f/4,\ f/4,\ f/4\right\}^t; \\ F_u^{(A_2)}(t) = P^{(A_2)} F_u(t) = \left\{0\right\}; \\ F_u^{(B_1)}(t) = P^{(B_1)} F_u(t) = \left\{0,\ f/4,\ -f/4,\ f/4,\ -f/4\right\}^t; \\ F_u^{(B_2)}(t) = P^{(B_2)} F_u(t) = \left\{0\right\}; \\ F_u^{(E^1)}(t) = P^{(E^1)} F_u(t) = \left\{0,\ f/2,\ 0,\ -f/2,\ 0\right\}^t; \\ F_u^{(E^2)}(t) = P^{(E^2)} F_u(t) = \left\{0\right\}. \end{gathered} \tag{6}$$

It can be seen that the sum of all found projections gives the original vector $F(t)$. From expressions (6), we can conclude that the considered force action excites three types of vibrations: longitudinal vibrations of the vehicle (irreducible representation A_1), longitudinal-bending vibrations of boosters (irreducible representation B_1) and bending vibrations of the rocket in the X_0Y_0 plane (1st line of irreducible representation E).

Conclusion. The symmetry analysis of the natural vibrations problem of the beam model of the clustered launch vehicle, considered in this article, led to results similar to [5–8]. At the same time, by virtue of its formality and mathematical rigor, this approach can be extended to the vibrations problem of the clustered launch vehicle with a different number (arbitrary) of boosters. The proposed method of reducing the multiple frequencies vibration modes to the rocket stabilization planes using simple operations represented by linear transformations can be useful in practical calculations. The orthogonal projection of the external perturbation vector on the subspaces of irreducible representations makes it possible to identify the types of the vehicle vibrations, excited by the considered vector.

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