VIBRATION DYNAMIC CHARACTERISTICS SIMULATION
FOR HELICOPTER MAIN GEAR BOX TEST BENCH

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Abstract
New elements are usually used in developing promising products, and they are introduced to improve its characteristics. This study objective is aimed at making certain decisions to eliminate or reduce negative effects of design and technological solutions that are being identified only at the bench testing stage. Problem of resonant interactions between bench elements and main gear box units is connected to significant distortion of the product under development dynamic analysis results. Therefore, identifying resonant interaction conditions and obtaining reliable results in the course of developing any kind of units is relevant for the aviation industry enterprises. JSC “Reduktor–PM” created a universal bench, and it was involved in solving this task practically for the first time, since significant oscillation levels were registered significantly exceeding the calculated values in the process of testing the bench. To determine the bench element base frequencies, the modal analysis task was finalized to identifying natural frequencies of the bench structural elements and partial frequencies of these elements at installing the unit on the bench. Wave transformation pattern in complex structures significantly depends on the element base and the structural connection between its components. In fact, the wave field structure is $n$-dimensional. Nevertheless, general approaches to generating interaction conditions are possible on the basis of their expansion in coordinates. In this case, the algorithm for determining oscillation natural frequencies in the bench elements and in the structure as a whole is based on the modal analysis.

Keywords
Dynamics, model, calculation, oscillations, vibrations, bench, gear box

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**Introduction.** Any mechanical transmission, including the test bench technological transmission, is a combination of many rotors with gears in bearing supports enclosed in load carrying structure casing. Existing methods in building mathematical models do not fully meet the requirements of solving the problem of simulating such systems in general and are rather appropriate in simulating separate elements that are not connected between each other. From the point of view of mathematical model, mechanical transmissions and mechanically closed benches for their testing are a combination of the complex geometry elements that are structurally interconnected. These connections have elastic and damping properties. Structural elements themselves are systems with distributed parameters. Works [1–7] are devoted to dynamic processes in gearing, works [8–10] — to planetary gear dynamics and works [11, 12] — to resilient connecting socket dynamics. Based on the most recent works devoted to transmissions dynamics, the problem of designing a dry friction damper for an aircraft engine gear box bevel gearwheel was considered [13]. The work was carried out in order to reduce the amplitude of wheel diaphragm vibration stresses to prevent its fatigue destruction. It should be noted that the literature review demonstrated insufficient level of scrutiny in the transmission dynamics problem and practically complete absence of works on benches for testing them.

Mechanically closed bench together with a gearbox under testing presents a closed transmission circuit. Structure of the scheme includes transmission shafts, gear wheel boxes, bearing supports, elastic couplings, housing units and load-bearing structure. This article is devoted to the problem of creating mathematical model of a similar complex system and its testing at the level of simplest test tasks having the analytical solution.

**Analytical problem statement in regard to coupled system forced oscillations.** When constructing a mathematical model of reducer — bench system, as well as of any transmission, at least two approaches are possible, i.e., using continuous systems with distributed parameters and discretization accompanied by building a system with multitude degrees of freedom. This article is devoted to construction of a model based on typical primitives with distributed parameters.

General structure of the distributed system oscillations equation has the following form:

$$A \frac{\partial^2 Y}{\partial t^2} + B \frac{\partial Y}{\partial t} + CY = f.$$  \hspace{1cm} (1)

Following notations are introduced in relation (1): \(Y(x, y, z, t)\) is the sought-for displacement function, generally spatial; \(C\) is stiffness operator; \(A\) is inertia
operator; $B$ is damping operator; $f(x, y, z, t)$ is external force impact on the system.

Equation (1) describes dynamic behavior of a separate structural element; however, the structure functioning as a whole involves interaction of elements with each other.

Complex structure could be represented as a system of separate elements; in this case, the main research problem appears in formulating conditions of communication between elements in the form of mathematical dependencies.

Mathematically elastic dependence between the $i$ and $j$ structural elements could be assumed as the transfer function:

$$K_{ijk} \delta(b_{ij})\left(Y_{jm}(b_{ji}, t)A_{ij}(m, k) - Y_{ik}(b_{ij}, t)\right),$$  \hspace{1cm} (2)

where $K_{ijk}$ is the force transfer coefficient (coupling rigidity); $\delta(b_{ij})$ is the $n$-dimensional unit pulse Dirac function; $b_{ij}, b_{ji}$ are the radius vectors of interface points on the $i$ and $j$ elements; $Y_{ik}(Y_{jm})$ is the $i(j)$ element motion in the $k$ ($m$) direction ($k$ ($m$) is the degree of freedom index); $A_{ij}(m, k)$ is the coordinate transformation matrix, which determines spatial orientation of the elements coordinate local systems.

Similarly, the damping transfer function could be determined as follows:

$$D_{ijk} \delta(b_{ij})\left(\frac{\partial Y_{jm}}{\partial t}(b_{ji}, t)A_{ij}(m, k) - \frac{\partial Y_{ik}}{\partial t}(b_{ij}, t)\right),$$  \hspace{1cm} (3)

where $D_{ijk}$ is the coupling damping coefficient.

Based on equations (1)–(3), the structure under study general system of dynamics equations could be formulated

$$A_{ik} \frac{\partial^2 Y_{ik}}{\partial t^2} + B_{ik} \frac{\partial Y_{ik}}{\partial t} + C_{ik} Y_{ik} =$$

$$= \sum_{jm} K_{ijk} \delta(b_{ij})\left(Y_{jm}(b_{ji}, t)A_{ij}(m, k) - Y_{ik}(b_{ij}, t)\right) +$$

$$+ \sum_{jm} D_{ijk} \delta(b_{ij})\left(\frac{\partial Y_{jm}}{\partial t}(b_{ji}, t)A_{ij}(m, k) - \frac{\partial Y_{ik}}{\partial t}(b_{ij}, t)\right) + f_{ik}. \hspace{1cm} (4)$$

Equation (4) appears to be a system of differential equations in partial derivatives.

In the general case, solution of such a problem is difficult and requires high degree of discretization, for example, using FEM with algebraic equation systems of ultrahigh dimension. However, system (4) could be reduced to a system of
ordinary differential equations applying the so-called generalized (main) coordinate method, and the displacement functions could be represented as:

$$Y_{ik} = \sum_{r=1}^{\infty} V_{ikr}(t) U_{ikr},$$

(5)

where $V_{ikr}(t)$ are the modal motion functions; $U_{ikr}$ are the eigenmodes (modes) of $i$-th component oscillations in the $k$ direction determined in the absence of dissipation; $r$ ($s$) is the oscillation form sequence number.

Equation (5) and the text further accept the following system of indexing the expression term: $i$ ($j$) first index is the structure under study element number; $k$ ($m$) second index is the degree of freedom number in the element local coordinate system; $r$ ($s$) third index is the modal coordinate number of the element given degree of freedom.

Based on (5) and taking into account the oscillation eigenmodes orthogonality property, and using the Bubnov — Galerkin method, equation (1) could be approximately represented as the $r \leq R$ system of ordinary differential equations of the following form:

$$\frac{d^2V_{ikr}(t)}{dt^2} + 2d_{ikr} \frac{dV_{ikr}(t)}{dt} + \omega_{ikr}^2 V_{ikr}(t) = F_{ikr},$$

(6)

where $d_{ikr}$ are the damping coefficients of corresponding oscillation modes; $\omega_{ikr}$ are the intrinsic cyclic frequencies of the $i$-th component oscillation in the $k$-direction determined in the absence of dissipation; $F_{ikr} = \frac{1}{M_{ikr}} \int_V f_{ikr} U_{ikr} dV$ is the generalized force; $M_{ikr} = \int_V \rho_i U_{ikr}^2 dV$ is the generalized mass; $\rho_i$ is the material density.

Applying representation (6) to the system of equations (4), final form of the dynamics equation system for the structure under study could be obtained:

$$\frac{d^2V_{ikr}(t)}{dt^2} + 2d_{ikr} \frac{dV_{ikr}(t)}{dt} + \omega_{ikr}^2 V_{ikr}(t) =$$

$$= \sum_{jm} K_{ijk} \frac{U_{imr}(b_{ij})}{M_{ikr}} \left( \sum_{s=1}^{R} V_{jmrs}(t) U_{jmrs}(b_{ij}) A_{ij}(m,k) - \sum_{r=1}^{R} V_{ikr}(t) U_{ikr}(b_{ij}) \right) +$$

$$+ \sum_{jm} D_{ijk} \frac{U_{ikr}(b_{ij})}{M_{ikr}} \left( \sum_{s=1}^{R} \frac{dV_{jmrs}(t)}{dt} U_{jmrs}(b_{ij}) A_{ij}(m,k) - \sum_{r=1}^{R} \frac{dV_{ikr}(t)}{dt} U_{ikr}(b_{ij}) \right) + F_{ikr}.$$

(7)
Equation (7) is a system of ordinary differential equations in regard to partial modal coordinates of the structure elements under study; solutions to this equation are the modal motion functions of the structure elements.

Solution to system (7) should be sought in the trigonometric Fourier series form

\[ V_{kr}(t) = \frac{a_0}{2} + \sum_{\nu=1}^{\infty} \left( a_\nu \cos (2\pi \nu t) + b_\nu \sin (2\pi \nu t) \right), \] (8)

where \( a_0, a_\nu \) and \( b_\nu \) are the calculated coefficients at harmonics; \( \nu \) is the harmonic carrier frequency \( (\omega = 2\pi \nu) \).

When substituting (8) in (7), the system of differential equations could be represented as a sequence of systems of linear algebraic equations (SLAE) in regard to the \( a_0, a_\nu \) and \( b_\nu \) coefficients. Harmonic frequencies number and carriers in solving system (7) are actually determined by the \( F_{kr} \) external generalized force harmonics composition, which could be represented in the form of Fourier series, similarly to (8). Free term present with the \( a_0 \) coefficient demonstrates possibility of accounting the static loading in solving the dynamic problems.

**Physical boundary and initial conditions.** Boundary conditions consideration in the system of equations (7) is quite simple; structure elements fixing conditions should be accounted even at the stage of determining their modal coordinates. Degree of freedom restrictions would be provided for only those elements that are connected to the structures, possess high rigidity, and which displacements are negligible. For example, these could be parts of the basement, where the test bench is located. Modal coordinates of structure elements conjugated with its other parts should be calculated with free boundary conditions that provide arbitrarily significant displacement along any of the degrees of freedom.

Initial conditions are determined by the partial initial phases of external harmonic effect. Thus, solution is a superposition of solutions for each external harmonic with its carrier frequency and phase that determines the initial conditions.

**Dynamic system element base structure of the object under study.** Differential equations of all elements of the structure under study, one way or another, could be represented in the form (1). Mathematical model developed has no restrictions in terms of using elastic models as an element base, since it is based on modal coordinates of the structure elements. However, calculating the natural oscillation modes of the complex shape fully three-dimensional objects is time-consuming from the point of view of software implementation or requires the use of third-party software based on the finite elements' method. As an example of the element base, beam and plate elastic models are provided.
Elastic beam model is most preferable for shafts and connecting springs. Both the Euler — Bernoulli model and the Timoshenko model could be used. In this work, the Timoshenko beam model [14] was applied, since the gear shafts often are having the length to diameter ratio of less than ten. Timoshenko model, in contrast to the Euler — Bernoulli model, takes into account shear strain and inertia of section rotation. This makes it possible to obtain a more accurate result for beams of any dimension. Similar mathematical model is preferable for beam elements of the bench supporting structure.

The beam could experience bending in two planes, i.e., tensile and torsion. In this case, modal coordinates of shafts, springs and beam supporting elements are determined by four independent dynamics equations.

Tension and torsion modal coordinates [15–18] (no. 1 and no. 2):

\[
\mu \frac{\partial y}{\partial t^2} - \frac{\partial}{\partial x}\left[EA \frac{\partial y}{\partial x}\right] = 0; \tag{9}
\]

\[
\bar{J} \frac{\partial \theta}{\partial t^2} - \frac{\partial}{\partial x}\left[GJ_p \frac{\partial \theta}{\partial x}\right] = 0, \tag{10}
\]

where \(\mu(x)\) is the beam linear mass; \(y(x)\) is the beam tension; \(E\) is the material rigidity module (Young); \(A(x)\) is the cross-section area; \(\bar{J}(x)\) is the beam linear moment of inertia; \(\theta(x)\) is the beam twist angle; \(G\) is the material shear module; \(J_p(x)\) is the section equatorial moment of inertia.

Modal bending coordinates in two planes [14] (no. 3 and no. 4):

\[
\rho A \frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial x}\left[kAG\left(\frac{\partial u}{\partial x} - \varphi\right)\right]; \tag{11}
\]

\[
\rho J \frac{\partial^2 \varphi}{\partial t^2} = \frac{\partial}{\partial x}\left(EJ \frac{\partial \varphi}{\partial x}\right) + kAG\left(\frac{\partial u}{\partial x} - \varphi\right),
\]

where \(\rho\) is the material density; \(A(x)\) is the cross-section area; \(u(x)\) is the beam deflection; \(k\) is the Timoshenko shear coefficient; \(G\) is the material shear module; \(\varphi(x)\) is the section tilt angle; \(J(x)\) is the section moment of inertia; \(E\) is the material rigidity module.

Gear disks are elements with two dominant sizes; therefore, a model of an elastic round plate based on the Kirchhoff — Love hypotheses in combination with the plate longitudinal deformation in the median plane would be a logical mathematical model for their description. Assembly casings in a first approximation could also be considered as a set of interconnected plates and shells. A plate may experience lateral bending and tension in the median plane. Thus,
discs and casing walls modal coordinates are determined by two independent
dynamics equations.

Transverse bending modal coordinates [15–18] (no. 1):

$$D \left( \frac{\partial^4 u}{\partial x^4} + \frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} \right) + \rho h \frac{\partial^2 u}{\partial t^2} = 0,$$

(12)

where $D = Eh^3 / 12 (1 - \gamma^2)$ is the cylindrical rigidity ($E$ is the material rigidity
module; $\gamma$ is the material Poisson’s ratio); $u$ is the plate deflection; $\rho$ is the
material density; $h$ is the plate thickness.

Longitudinal oscillations modal coordinates (no. 2):

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{2} (1 - \gamma) \frac{\partial^2 u}{\partial y^2} + \frac{1}{2} (1 + \gamma) \frac{\partial^2 v}{\partial x \partial y} = \rho \frac{(1 - \gamma^2)}{E} \frac{\partial^2 u}{\partial t^2};$$

$$\frac{1}{2} (1 - \gamma) \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{1}{2} (1 + \gamma) \frac{\partial^2 u}{\partial x \partial y} = \rho \frac{(1 - \gamma^2)}{E} \frac{\partial^2 v}{\partial t^2},$$

(13)

where $u(x, y)$ are the displacements along the $x$-axis; $v(x, y)$ are the
displacements along the $y$-axis; $\rho$ is the material density; $\gamma$ is the material
Poisson’s ratio; $E$ is the material rigidity module.

Bearing supports and elastic couplings are characterized primarily by their
rigidity and damping properties, their inertial component could be considered
as an absolutely solid body.

**Test problems.** Timoshenko beam elastic model is considered in this article
as an element base in the test problems. To implement the mathematical model,
MATLAB mathematically oriented programming language was used.

The problem of free oscillations for a composite round beam on two hinged
supports was solved as the first test problem in order to verify the mathematical
apparatus functioning and its implementation algorithm (Fig. 1). The beam was
assembled of five segments having the same length of 0.2 m, the beam total
length was 1 m; segment diameter 25 mm; material properties (steel, rigidity
module $2 \cdot 10^{11}$ Pa; density $7,850$ kg/m$^3$).

**Fig. 1. Test modal problem calculation scheme**

Composite beam calculation results should correspond to the solid structure of
similar size and shape. Analytical solution of the modal problem for this
calculation scheme is well known [15, p. 260]. Natural frequencies are determined by the following formula

$$p_i = \frac{i^2 \pi^2}{l^2} \sqrt{\frac{EJ}{\mu}},$$  \hspace{1cm} (14)$$

where $p_i$ is the frequency with serial number $i$; $l$ is the beam length; $E$ is the rigidity module; $J$ is the section moment of inertia; $\mu$ is the section specific gravity.

Table 1 presents comparative results of solving the modal problem.

<table>
<thead>
<tr>
<th>Serial number frequency $i$</th>
<th>Mathematical model</th>
<th>Analytic calculation</th>
<th>Error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>49.3205/49.3205 Hz</td>
<td>49.5542 Hz</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>197.3634/197.3634 Hz</td>
<td>198.2166 Hz</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>442.7554/442.7549 Hz</td>
<td>445.9874 Hz</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>788.2172/788.2174 Hz</td>
<td>792.8665 Hz</td>
<td>0.6</td>
</tr>
</tbody>
</table>

Results presented in Table 1 demonstrate the first eight pair of segmented model’s bending eigenfrequencies in two planes; calculation error relative to the analytical calculation of bending frequencies is less than 1 %. The result was obtained using 14 modal coordinates for the degree of freedom.

As the second test problem, let us consider oscillations of a similar composite round beam on two hinged supports loaded with induced harmonic force. Design scheme is presented in Fig. 2.

![Fig. 2. Forced oscillations test problem design scheme](image)

The beam is loaded with the $P(t) = P_0 \cos \omega t$ harmonic force. Analytical solution to the beam deflection problem at its center is known [16, p. 166]. deflection amplitude value is determined by the following formulas:

$$u\left(\frac{l}{2}\right) = \frac{P_0 l^3}{32 EJ} \frac{1}{\lambda^3} (\tan \lambda - \theta \lambda);$$  \hspace{1cm} (15)$$
\[ \lambda = \frac{1}{2} \sqrt[4]{\mu \omega^2 l^4 / (E J)} \]  

Here \( P_0 \) is the force amplitude; \( l \) is the beam length; \( E \) is the rigidity module; \( J \) is the section moment of inertia; \( \mu \) is the section specific gravity; \( \omega \) is the force angular frequency.

Table 2 shows results of the displacement calculation in the plane of the \( P_0 = 1000 \) N force applied at various carrier frequencies. The result was obtained using 14 modal coordinates for the degree of freedom.

<table>
<thead>
<tr>
<th>Frequency, Hz</th>
<th>Mathematical model</th>
<th>Analytic calculation</th>
<th>Error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.0065 m</td>
<td>0.0065 m</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>0.0158 m</td>
<td>0.0154 m</td>
<td>2</td>
</tr>
<tr>
<td>60</td>
<td>0.0112 m</td>
<td>0.0114 m</td>
<td>1.8</td>
</tr>
<tr>
<td>90</td>
<td>0.0022 m</td>
<td>0.0022 m</td>
<td>0</td>
</tr>
<tr>
<td>120</td>
<td>0.0010 m</td>
<td>0.0010 m</td>
<td>0</td>
</tr>
</tbody>
</table>

Errors in calculating deflection in the beam center at the presented carrier frequencies did not exceed 2%.

**Conclusion.** Developed mathematical model shows a very high convergence with the well-known analytical solution; the segmented beam works as a whole. It should be noted that the Timoshenko theory was used in the mathematical model for the beams’ element base, while analytical solutions were obtained for the classical Euler — Bernoulli beam model, which also imposes certain errors when comparing the results.

Mathematical model obtained uses the element base necessary in describing the structure and is a rather flexible tool in describing system dynamics of any complexity. Besides, it is able to solve the problem of simulating a multi-stage helicopter transmission and such a problem as gear — bench system determining the mechanism system components interaction. With sufficient level of automation, it is possible to obtain an efficient design technique for dynamically stable multi-stage transmission systems. Technique under development would significantly improve the design process of helicopter transmissions and their test benches, as the main products of JSC “Reduktor-PM”.

Experimentally verified mathematical model of this kind would make it possible to solve the practical problem of determining the influence of the gearbox under testing and of the test bench on the characteristics of the gear-
bench system as a whole, as well as the mutual influence of these subsystems on each other.

Next stage of scientific work performed by the authors would be building a mathematical model of the operating test bench comprising helicopter gearbox under testing and its verification with the obtained experimental data.

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